is high (≈2,000). Rivulets can meander at values of I far less than unity when the liquid viscosity is high and the corresponding Reynolds number is low, apparently through a mechanism entirely different from that envisioned here for water rivulets. Culkin (1981) finds that rivulets of water-glycerine mixtures meander soon after large amplitude travelling waves appear on the free surface. The resulting motion of the contact lines is described and quantitative data for the critical flow rate to produce meandering are presented.

It is interesting to note that the high Reynolds number analysis of the stability of water rivulets succeeds because of the presence of significant contact angle hysteresis. The hysteresis force stabilizes the position of the contact lines and prevents the usual linear instability (Davis, 1980). Only when the hydrodynamic disturbances grow to sizes sufficient to overcome hysteresis can the contact lines move, and this seems to occur at high Reynolds number in the case of water rivulets.

We have shown that under carefully controlled conditions leading to rivulet formation, the critical flow rate at which spontaneous meandering begins (Q-from-below) and the critical flow rate needed to sustain unsteady meandering (Q-from-above) are reproducible and well defined. These useful concepts lead to a simple correlation between the behavior of water rivulets upon a variety of materials and a single dimensionless number, the stability parameter I. For many engineering applications, this correlation provides a simple means by which the gross flow characteristics of water rivulets can be estimated, given the liquid materials properties, and advancing and receding contact angles, and the inclination of the solid. In systems with large contact-angle hysteresis, much larger flow rates are required to produce and sustain unsteady meandering, and the error of estimation for the exact transition point becomes large.

ACKNOWLEDGMENT

This work was supported by the National Science Foundation, Fluid Mechanics Program.

NOTATION

$\mathcal{A}$ = surface area
$\mathcal{C}$ = surface contour
H = hysteresis index defined in Eq. 2
I = stability index defined in Eq. 1
$\bar{I}$ = average value of I
Q = flow rate
s = arc length
u = axial velocity in straight rivulet

Greek Letters

β = inclination angle of plate
ρ = density of liquid
σ = interfacial tension
θ = contact angle
θ_A = advancing contact angle
θ_R = receding contact angle

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Manuscript received December 6, 1982; revision received June 3, and accepted June 23, 1983.

Experimental Study of Deep Bed Filtration: A Stochastic Treatment

A stochastic pure birth process, which describes pore blockage in a filtration process, has been coupled with the Carman-Kozeny equation to simulate the pressure buildup in deep bed filtration. It has been shown that the resultant one parameter model equation fits adequately the experimental data obtained under the straining dominated condition, and that available data obtained under the adhesion dominated condition can also be described by the model.

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SCOPE

Granular filters have long been used to remove suspended solids from water in both the purification of potable water and the treatment of waste water (Fuller, 1898; Dunbar and Calvert, 1908; Hall, 1957). The suspended solids are retained in the bed either through a straining mechanism or an adhesion mechanism (see, e.g., Tien and Payatakes, 1979). The straining mechanism dominates when the characteristic diameter of the suspended solids is large relative to the pore diameter in the bed matrix. Conversely, the adhesion mechanism dominates when the characteristic diameter is much smaller than that of the pore. Adhesion of the suspended solids to the collector particles takes

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place when the two come into contact.

Mathematical models of the granular or deep bed filtration process can be categorized into three classes: those based solely on the phenomenological equation of kinetics (Iwasaki, 1937; Ives, 1960; 1961; Camp, 1964); those derived through the trajectory analysis (O'Melia and Stumm, 1967; Yao et al., 1971; Payatakes, 1973; Rajagopalan and Tien, 1976; 1979; Pendse et al., 1978); and those derived through the stochastic approach (Litwiniszyn, 1963, 1966, 1967, 1968a,b, 1969).

Stochastic models can be viewed as the intermediate between the phenomenological models in which the form of model equations is assumed from prior knowledge of the phenomenon and the trajectory analysis models in which the trajectories of the particles are painstakingly determined from the force balance equation. The stochastic models, derived through probability considerations, often generate parameters that are easier to identify. At the same time, the solution of the resulting model equations can be accomplished without undue effort. The filtration process has been modeled as a pure birth process (Litwiniszyn, 1963), a birth-death process (Litwiniszyn, 1966), a random walk process (Litwiniszyn, 1967), and state and time continuous Markov processes, i.e., stochastic diffusion processes (Litwiniszyn, 1968a,b, 1969). The works of Litwiniszyn have not received sufficient recognition mainly due to the failure in relating the resultant models to measurable variables. For example, in his pure birth model, the number of blocked pores is considered as the model variable, and in his birth-death model, the number of trapped particles is considered as the model variable. These variables can not be determined readily through conventional experimental techniques.

In the present work, the pure birth process is adopted to model the straining dominated filtration process. Model variables, however, have been related to a measurable variable to facilitate comparison with experimental data.

CONCLUSIONS AND SIGNIFICANCE

A stochastic model, namely, the pure birth model, which was first employed by Litwiniszyn (1963) to describe pore blockage in a filtration process, has been modified by incorporating the Carman-Kozeny equation to simulate the pressure buildup during constant-flow filtration. The model, containing one adjustable parameter, the blockage constant, appears to describe adequately the pressure drop data for both straining dominated and adhesion dominated filtration processes. Among factors affecting the blockage constant are size and distribution of the collector grains and the suspended solids, slurry concentration, bed porosity, and filtration rate. Establishment of an empirical relationship between the blockage constant and the pertinent variables should provide necessary information predicting pressure buildup in filtration. The shortcoming of the model is that it does not simulate the concentration profile of suspended or deposited solids in the filter.

MODEL

A stochastic process $\{N(t);t\epsilon(0,\infty)\}$ is a family of random variables describing a process or phenomenon whose development is governed by probabilistic laws (see, e.g., Chiang, 1968). In considering the filtration process as a stochastic process, the number of blocked pores, $N(t)$, at time t can be taken as the random variable (Litwiniszyn, 1963); a specific value of $N(t)$ will be denoted by n. For the pure birth process, it is considered that given $N(t) = n$ (Chiang, 1968),

1. the conditional probability that a new event will occur, i.e., that an open pore will be blocked during the interval $(t, t + \Delta t)$, is $\lambda_n \Delta t + o(\Delta t)$, where λ_n is a function of n, and
2. the conditional probability that more than one event will occur in this time interval is $o(\Delta t)$.

The notation $o(\Delta t)$ signifies that

$$\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$$

Obviously, the probability of no change in the interval $(t,t + \Delta t)$ is $[1 - \lambda_n \Delta t - o(\Delta t)]$. Under the assumptions of the pure birth process, the possibility of the occurrence of scouring is neglected. The probability that there are n pores blocked at the moment t is denoted by $P_n(t) = Pr\{N(t) = n\}, n = 0,1, \ldots$ Suppose that $(0,t)$ and $(t,t + \Delta t)$ are two successive time intervals. Then, the occurrence of exactly n pores being blocked during the time interval $(0,t + \Delta t)$ can be realized in three mutually exclusive ways:

1. All n pores are blocked in $(0,t)$ and none in $(t,t + \Delta t)$, with the probability of such occurrence written as $p_n(t)[1 - \lambda_n \Delta t - o(\Delta t)]$.
2. Exactly $(n - 1)$ pores are blocked in $(0,t)$ and one pore is blocked in $(t,t + \Delta t)$, with probability $p_{n-1}(t)[\lambda_{n-1}\Delta t + o(\Delta t)]$.
3. Exactly $(n - j)$ pores are blocked in $(0,t)$ and j pores are blocked in $(t,t + \Delta t)$ where $2 \le j \le n$, with probability $o(\Delta t)$.

By taking into account all these possibilities and combining all quantities of order $o(\Delta t)$, the following is obtained (Chiang, 1968):

$$p_n(t + \Delta t) = p_n(t)[1 - \lambda_n \Delta t] + p_{n-1}(t)\lambda_{n-1}\Delta t + o(\Delta t), n \ge 1 \quad (1)$$

and

$$p_0(t + \Delta t) = p_0(t)[1 - \lambda_0 \Delta t] + o(\Delta t) \quad (2)$$

Rearranging these equations and taking the limit as $\Delta t \to 0$ yield

$$\frac{dp_n(t)}{dt} = -\lambda_n p_n(t) + \lambda_{n-1} p_{n-1}(t), n \ge 1 \quad (3)$$

and

$$\frac{dp_0}{dt} = -\lambda_0 p_0(t) \quad (4)$$

Litwiniszyn (1963) has assumed that the intensity of transition, λ_n, takes the form

$$\lambda_n = k(n_0 - n), \quad n = 0,1,2, \ldots, n_0 \quad (5)$$

where n_0 is the total number of open pores at the moment $t = 0$ susceptible to blockage and k is a proportionality constant. Equation 5 implies that the rate of pore blockage is proportional to the number of open pores. The constant k may be called the blockage constant. Introducing Eq. 5 into Eqs. 3 and 4, the following equations are obtained;

$$\frac{dp_n(t)}{dt} = -k(n_0 - n)p_n(t) + k[n_0 - (n - 1)]p_{n-1}(t), \quad n = 1,2, \ldots, n_0 \quad (6)$$

$$\frac{dp_0(t)}{dt} = -kn_0 p_0(t) \quad (7)$$

At the start of the filtration process, all pores are open; thus, the initial conditions to Eqs. 6 and 7 may be expressed as

$$\left.\begin{aligned} p_0(0) &= 1 \\ p_n(0) &= 0 \quad n = 1,2,\ldots,n_0 \end{aligned}\right\} \tag{8}$$

Solutions of Eqs. 6 and 7, subject to the initial conditions, Eq. 8, yields the distribution of the probabilities (Litwiniszyn, 1963). However, such information has limited practical implication; it is more meaningful to determine the average number or the expected value of blocked pores at a given moment t. By definition the expected value of the random variable, $N(t)$, is

$$E[N(t)] = \sum_{n=0}^{\infty} n p_n(t) \tag{9}$$

Evaluation of the righthand side can be conveniently accomplished by resorting to the probability generating function defined as

$$G(s,t) = \sum_{n=0}^{\infty} s^n p_n(t) \tag{10}$$

Multiplying both sides of Eq. 6 by the respective s^n's and Eq. 7 by s^0, and summing all the resultant equations, the following expression is obtained upon rearrangement.

$$\frac{\partial G(s,t)}{\partial t} = kn_0(s-1)G(s,t) - ks(s-1)\frac{\partial G(s,t)}{\partial s} \tag{11}$$

The initial conditions, expressed in Eq. 8, are transformed to

$$G(s,0) = 1 \tag{12}$$

Solution of Eq. 11 with the initial condition Eq. 12 yields (Litwiniszyn, 1963)

$$G(s,t) = [s - (s-1)e^{-kt}]^{n_0} \tag{13}$$

Comparison of Eqs. 9 and 10 indicates that

$$E[N(t)] = \left.\frac{\partial G(s,t)}{\partial s}\right|_{s=1} \tag{14}$$

Substitution of Eq. 13 into Eq. 14 yields

$$E[N(t)] = n_0(1 - e^{-kt}) \tag{15}$$

Similarly, the variance of the random variable, $N(t)$, may be obtained with the formula

$$\mathrm{Var}[N(t)] = \left.\left\{\frac{\partial^2 G(s,t)}{\partial s^2} + \frac{\partial G(s,t)}{\partial s} - \left[\frac{\partial G(s,t)}{\partial s}\right]^2\right\}\right|_{s=1} \tag{16}$$

Substitution of Eq. 13 into this formula gives

$$\mathrm{Var}[N(t)] = n_0 e^{-kt}(1 - e^{-kt}) \tag{17}$$

To utilize the results obtained above, it is necessary to relate the expected value of the number of blocked pores to measurable variables. For the case of constant rate filtration, it is assumed that all pores are of the same radius, r. Then, the flow rate of slurry through the filter can be expressed as

$$q = \{n_0 - E[N(t)]\}\pi r^2 v(t) \tag{18}$$

where $v(t)$ is the linear velocity of the flow in the pores at the moment t. Denoting the velocity of slurry flowing through the pores at the onset of the filtration process as v_0, it follows that

$$\begin{aligned} q &= n_0 \pi r^2 v_0 \\ &= \{n_0 - E[N(t)]\}\pi r^2 v(t) \end{aligned} \tag{19}$$

or upon rearrangement,

$$v(t) = \frac{n_0}{n_0 - E[N(t)]} v_0 \tag{20}$$

The pressure drop for laminar flow through a packed bed can be adequately described by the Carman-Kozeny equation (see, e.g., Zenz and Othmer, 1960)

$$\frac{\Delta P}{L} = 150 \frac{(1-\epsilon)^2}{\epsilon^3} \frac{\mu u}{d_p^2} \tag{21}$$

The value of 150 (±50%) in Eq. 21 is an average for beds of various particle shapes. It is valid only for incompressible beds with porosities in the range of 0.35 ~ 0.5. The superficial velocity, u, in the Carman-Kozeny equation can be expressed in terms of the linear velocity, v, as

$$u = \epsilon v \tag{22}$$

Substituting Eq. 22 into Eq. 21 yields

$$\frac{\Delta P}{L} = 150 \left(\frac{1-\epsilon}{\epsilon}\right)^2 \frac{\mu v}{d_p^2} \tag{23}$$

However, the change in the linear fluid velocity through the pores during the course of a filtration run is expressed by Eq. 20. Thus, substituting Eq. 20 into Eq. 23 results in

$$\frac{\Delta P(t)}{L} = 150 \left(\frac{1-\epsilon}{\epsilon}\right)^2 \frac{\mu v_0}{d_p^2} \left\{\frac{n_0}{n_0 - E[N(t)]}\right\} \tag{24}$$

The expression for $E[N(t)]$, Eq. 15, can now be substituted into Eq. 24 to obtain

$$\frac{\Delta P(t)}{L} = 150 \left(\frac{1-\epsilon}{\epsilon}\right)^2 \frac{\mu v_0}{d_p^2} e^{kt} \tag{25}$$

For a straining dominated filtration process, it can be assumed that the blockage of the pores does not affect the porosity downstream in the bed (Payatakes, 1973). In other words, for simplicity, it is assumed that an increase in the pressure drop can be attributed only to the increase in the linear velocity of flow. Equation 25, therefore, can be written as

$$\frac{\Delta P(t)}{L} = \left(\frac{\Delta P}{L}\right)_0 e^{kt} \tag{26}$$

where $(\Delta P/L)_0$ is a constant and reflects the initial pressure drop through the filter.

An alternate form of Eq. 26 has been proposed by Hermans and Bredée (1936) through a deterministic approach. They considered four types of filtration law: cake, intermediate blocking, standard blocking, and complete blocking. All four filtration laws can be derived from a single differential equation expressed as (Grace, 1956)

$$\frac{d^2 t}{dV^2} = K\left(\frac{dt}{dV}\right)^m \tag{27}$$

where K and m are empirical parameters. Setting $m = 0, 1, 1.5$, and 2 reduces Eq. 27 to the cake filtration law, intermediate blocking law, standard blocking law, and complete blocking law, respectively. While three of these laws can be derived by assuming appropriate filtration mechanisms (Hermans and Bredée, 1936; Grace, 1956), the intermediate blocking law cannot be derived mechanistically and is strictly empirical. For example, the cake filtration law can be derived by considering the cake as a series of parallel Poiseuille capillary tubes of increasing length. For constant rate filtration, Grace (1956) has shown that the intermediate blocking law can be expressed as

$$\frac{1}{q_0} \ln\left(\frac{\Delta P}{\Delta P_0}\right) = K_i t \tag{28}$$

which is equivalent to Eq. 26. However, working with thin (fabric type) filter media, Grace considered the empirical intermediate blocking filtration law to be insignificant.

FACILITY AND METHODS

A schematic of the experimental setup is shown in Figure 1. The setup comprises an upflow deep bed filter, slurry and water tanks, pumps, pressure transducer, transducer indicator, and a strip chart recorder. The plexiglass filter had an inside diameter of 0.132 m (5³⁄₁₆ in.) and a length of 0.457 m (18 in.) between the bottom distributor plate and the outlet. The filtering medium consisted of $-\#20 + \#30$ (20 × 30) mesh silica sand with a density of 2,600 kg/m^3 and an average diameter of approximately 710 μm. The

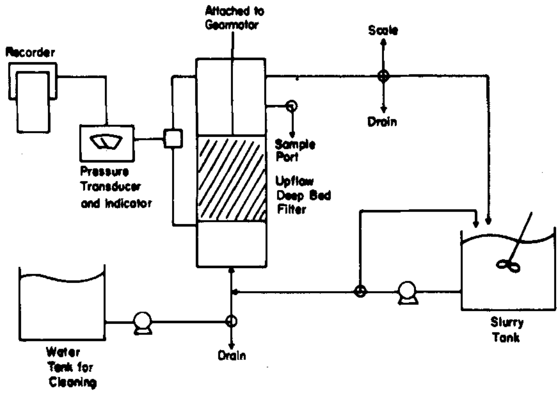

Figure 1. Experimental setup.

bed of sand, between 0.127 m (5 in.) and 0.330 m (13 in.) in depth, was supported between a bottom distributor plate and a top moveable porous plate. These plates retained the sand particles but allowed the slurry to flow through. The slurry was composed of sub #50 mesh coal particles with a density of 1,540 kg/m^3 dispersed in water. The distribution of the coal particle size was approximated by a log-normal distribution (Smith and Jordan, 1964) with a geometric mean characteristic length of 177 μm and geometric standard deviation of 1.97. The concentration of the slurry ranged from 0.01 to 0.1 wt. %. The coal particles were maintained in suspension in the slurry tank by agitation with a $\frac{1}{4}$ Hp (187 W) twin propeller mixer.

The porosity of the bed was dependent on the packing conditions; it ranged from 0.43 under the least dense static condition to 0.39. Prior to initiation of the filtration run, the sand was allowed to settle and form a bed with a porosity of about 0.40 to 0.41. The top porous plate was then placed and maintained at a position immediately above the sand. At the onset of the filtration run, slurry from the slurry tank was introduced at a constant rate of 48.8 m/h to the bottom of the filter by a HYPRO piston pump. The slurry was then allowed to flow upward through the filter. The pressure buildup in the filter was monitored with a variable reluctance pressure transducer which was connected to pressure taps positioned below the bottom distributor plate and near the outlet of the filter, Figure 1. Samples of the filtrate were collected intermittently from a sample port located between the upper porous plate and the outlet. These samples were then filtered in a 200 mL pressure filter to determine the solids content. The filtration run was terminated when the pressure drop through the filter reached 124 kPa (18 psi).

The filter was cleaned by fluidizing the entire bed with water to remove the deposited solids. During this period, the upper porous plate was raised to a position well above the fluidizing sand particles. The coal particles which did not stick to the sand were easily flushed from the filter in this manner. With the bed regenerated, the filtration cycle could be reinitiated.

RESULTS AND DISCUSSION

Figure 2 shows typical data of pressure drop encountered during the course of a filtration run plotted in the manner of Eq. 26. The buildup of pressure in the filter was extremely sensitive to the initial porosity of the bed; thus, the data shown in the figure represent the average of at least four experimental runs. The stochastic pure birth model appears to adequately fit the pressure drop data. However, for the system studied, there existed an initial period in which no significant pressure buildup was measurable. This was especially apparent in the filtration of the 0.01 wt. % coal slurry (Figure 2). Such a delay in headloss increase is not predicted by the model. The occurrence of this initial delay in the pressure buildup can probably be attributed to the fact that the slurry contained widely distributed sizes of coal particles; approximately 40% by volume of the coal

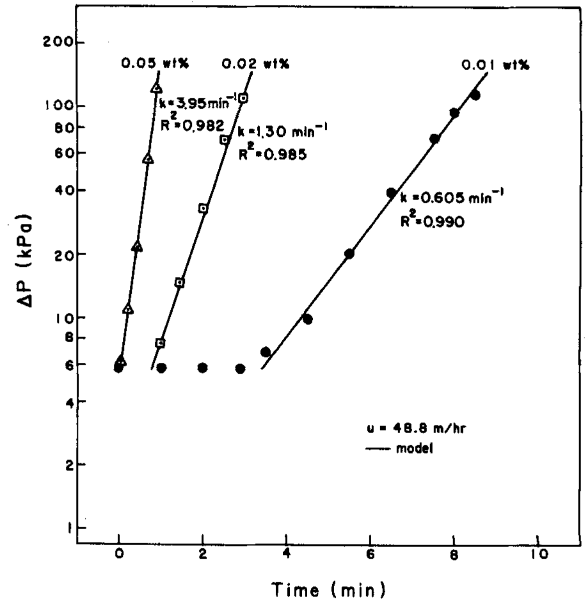

Figure 2. Pressure drop as a function of filtration time with slurry concentration as the parameter.

particles was smaller than two-tenths of the sand grain diameter, the size generally considered to be the lower bound for straining dominated capturing mechanism (Maroudas and Eisenklam, 1965a; Tien and Payatakes, 1979). Thus, it may be postulated that during the initial period, the pore openings were too wide to capture a significant portion of the suspended particles. As the larger particles were deposited, however, the pore size distribution in the bed gradually shifted toward the state where some of the smaller particles could also be captured by straining. Such an explanation appears plausible in view of the fact that published data of filtration of unisize or near size particles do not exhibit this type of behavior.

The concentration of suspended particles in the filtrate or effluent, C_e, is shown as a function of filtration time in Figure 3. It appears that the removal of solids from the liquid stream was independent of the slurry concentration and the filtration time, or in other words, the condition of the filter. Around 88% of the solids was removed.

The model equation, Eq. 26, contains one adjustable parameter, k. The initial pressure drop, $(\Delta P/L)_0$, may be estimated with the use of the Carman-Kozeny equation. The pore blockage constant, k, is affected by many factors, among which are size and distribution of collector grains and suspended particles, properties of the liquid and involved surfaces, filtration velocity, bed porosity and slurry concentration. For the system studied, the depth of the filter did not seem to affect the filtration process. Neglecting the

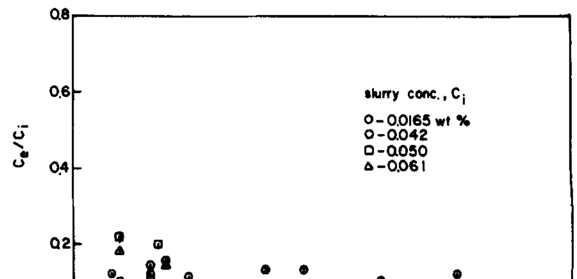

Figure 3. Concentration of suspended particles in the effluent, C_e, during the course of the filtration run.

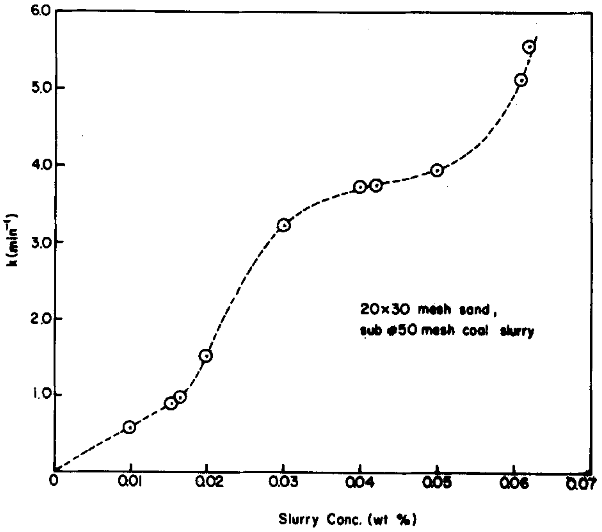

Figure 4. Dependence of the blockage constant, *k*, on slurry concentration.

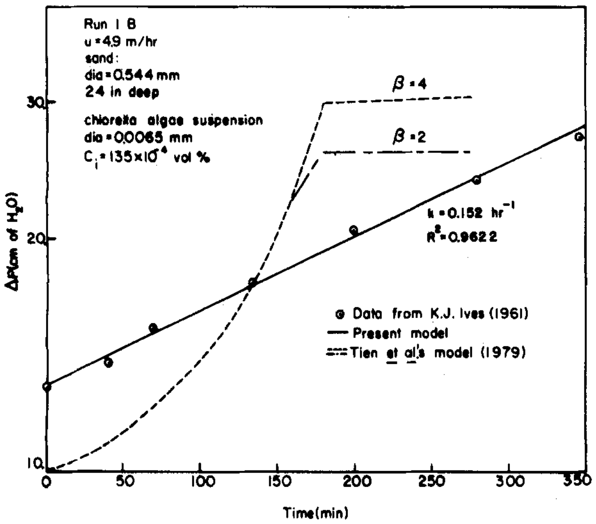

Figure 5. Comparison of Ives' (1961) data to the present model and the model of Tien et al. (1979).

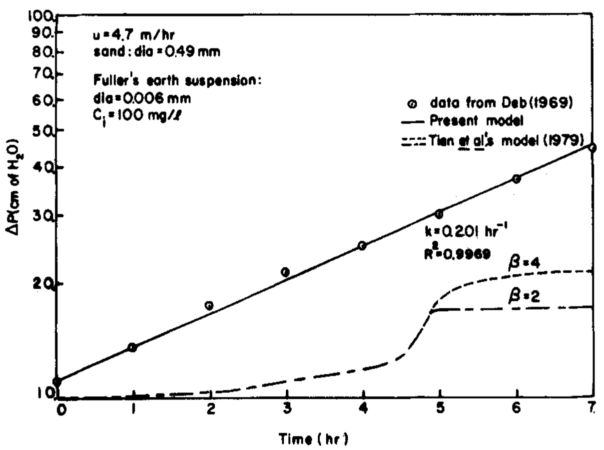

Figure 6. Comparison of Deb's (1969) data to the present model and the model of Tien et al. (1979).

initial delay period, the blockage constant may be obtained through a linear regression analysis of the time dependent pressure drop data. The dependence of the blockage constant, k, on the slurry concentration is depicted in Figure 4. As stated previously, the buildup of pressure in the filter was quite sensitive to the initial

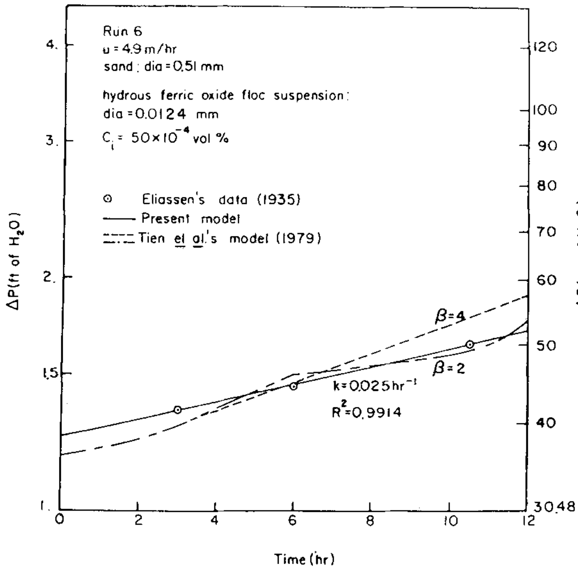

Figure 7. Comparison of Eliassen's (1935) data to the present model and the model of Tien et al. (1979).

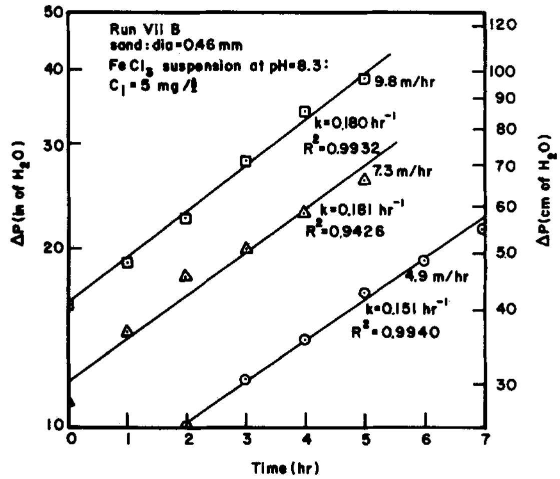

Figure 8. Comparison of Rimer's (1968) data to the present model.

porosity of the bed. Despite the care taken in the preparation of the filter bed, considerable variation in the rate of pressure buildup was observed. A standard deviation of approximately 30% of the k value was typically obtained at a given slurry concentration.

Equation 26 has been derived for a straining dominated filtration process in which scouring of deposited particles is negligible. However, as shown in Figures 5 through 8, the model equation can be fitted to data from a filtration process where adhesion was the dominating capturing mechanism. A basic assumption made in the derivation of Eq. 26 was the constancy of the bed porosity. It is generally thought that such an assumption may not be valid for an adhesion dominated filtration process. Works of Ives (1961) and Deb (1969) provided the necessary insights in determining the magnitude of the change in bed porosity as the filtration process proceeds. Using algae grown in radioactive nutrient medium as the suspended solid, Ives was able to determine directly the specific deposit, σ, at different depths in the filter without disturbing the process during a filtration run; a scintillation counter was used to detect γ-radiation from the algae. His measurements showed that at the end of a filtration run, when algae deposition was at a maximum, the specific deposit at a depth of $0.1L$ was less than 0.02. The specific deposit at a depth of $0.3L$ was essentially zero. Deb (1969), on the other hand, calculated the specific deposit from the measured free stream concentration of suspended solids at different

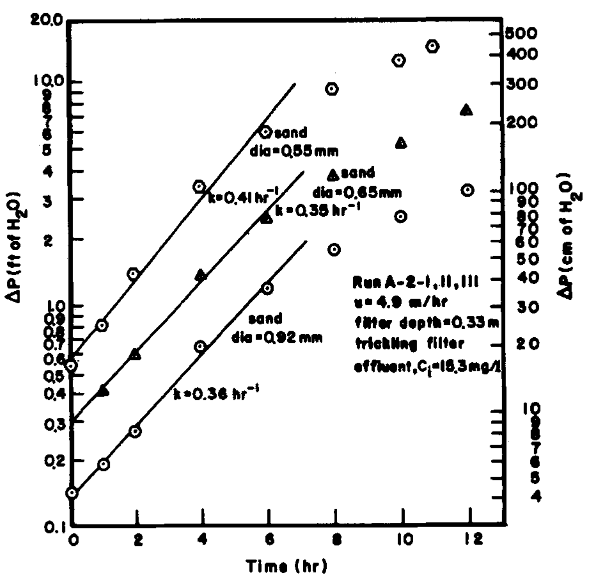

Figure 9. Comparison of Huang's (1972) data to the present model.

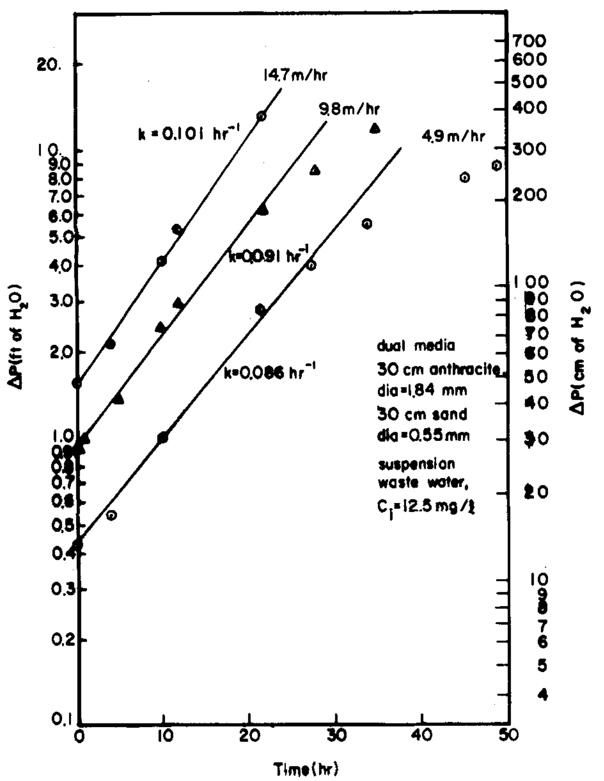

Figure 10. Comparison of Huang's (1972) dual media filter data to the present model.

depths in the filter. Similar to Ives' results, Deb's calculations showed that the values of the specific deposit were approximately 0.02 at a depth of $0.1L$ and 0.01 at the depth of $0.12L$ toward the end of the run. Thus, these earlier works showed that no appreciable change in bed porosity occurs in at least 80% of the deep bed filter. The studies by Ives and Deb were performed with 0.61 m deep sand filters. It is conceivable that for deeper beds, as much as 90 to 95% of the bed may have negligible change in porosity due to shallow penetration of the suspended solids. It now appears that the assumption of negligible change in overall porosity of the bed may also be adequate for an adhesion dominated filtration process. This may be the key to the good fit observed between the model and the data obtained from sand (Figures 5 through 9), dual media

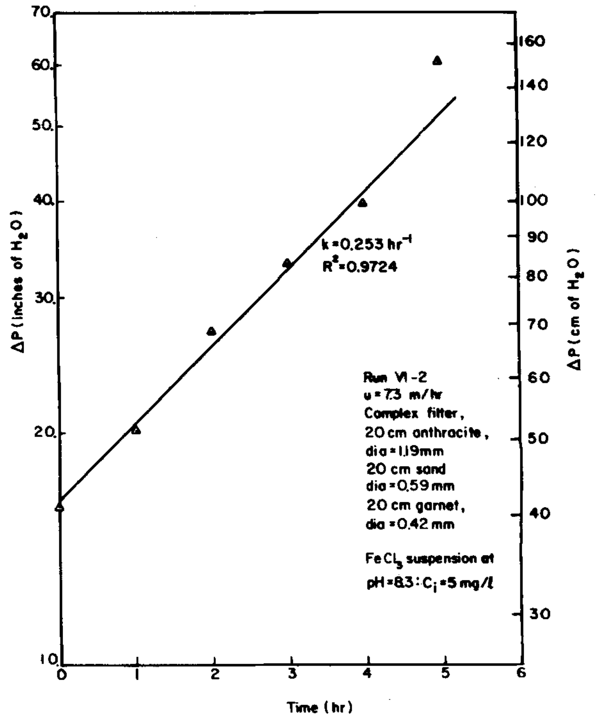

Figure 11. Comparison of Rimer's (1968) complex filter data to the present model.

(Figure 10), and complex filters (Figure 11). In Figures 5 through 7, pressure buildup predicted by the constricted tubes model of Tien et al. (1979) is also plotted for comparison. The parameter β in Tien et al.'s model is a correction factor for the volume of deposited matter necessary to block one unit cell in the filter. The dependence of the blockage constant, k, on the filtration rate can be detected in Figures 8 and 10, while its dependence on the collector grain size can be seen in Figure 9.

The data of Huang (1972), which appear in Figures 9 and 10, show a departure from the predicted behavior in the later stage of the filtration runs. This normally indicates redistribution of captured particles by scouring in the filter (Maroudas and Eisenklam, 1965b); however, no deterioration of filtrate quality was reported. Such departure also may be due to two other factors: biological cell decay and/or significant change in the bed porosity. The work of Huang involved filtration of a trickling filter secondary effluent in which suspended solids consisted mainly of microbial cells. Secondary effluent usually contains little or no metabolizable organics. Under such a condition, biological or microbial cell decay, due to endogenous respiration, cell lysis, or predation by higher organisms, could be very significant and could result in a reduction in the cell mass. (See, e.g., Rich, 1963.) It is possible that in Huang's study, especially the data shown in Figure 10, the filtration process lasted long enough to have an extensive decay of the deposited biomass. Such a decay would reduce specific deposit, and thus, retard the increase in the pressure drop. It is also possible that the assumption of negligible change in bed porosity was not valid in Huang's work. The data presented in Figure 9 were obtained with sand filters of 0.33 m in depth or about *half the size of filters* employed by Ives (1961) and Deb (1969). If we assume that, as in Ives and Deb's works, the penetration into the filter is approximately 0.2(0.61 m) = 0.12 m, then this would account for about 40% of the filter in which the change in bed porosity is significant. Such an estimate may be considered conservative since the grain size of the sand used in Huang's work (Figure 9) was larger than that used by Ives and Deb and it is also possible that because of redistribution of the captured particles in the bed, the blocked capillary tubes may reopen again, and hence decrease the rate of pressure drop. Therefore, the occurrence of biological cell decay and/or a significant change in the bed porosity and/or the redistribution of the captured particles by scouring could very well account for the differences between the experimental data and model prediction.

ACKNOWLEDGMENT

We are grateful to Department of Energy (Grant No. FG01-78ET12230) and Engineering Experiment Station, Kansas State University (Kansas Energy Study Project) for financial assistance.

NOTATION

C_e	= concentration of suspended solids in the filtrate
C_i	= slurry concentration
d_p	= average diameter of a grain of sand
$E\{N(t)\}$	= expected value of the random variable $N(t)$
$G(s,t)$	= probability generating function
K	= empirical parameter in Eq. 27
K_i	= plugging constant of the intermediate blocking law
k	= proportionality constant defined in Eq. 5, the blockage constant
L	= depth of the filter
m	= empirical parameter in Eq. 27
$N(t)$	= random variable which describes the number of blocked pores at the moment t
n	= values which may be assumed by the random variable $N(t)$
n_0	= total number of open pores susceptible to blockage
$p_n(t)$	= probability that there are n pores blocked at the moment t
$\Delta P/L$	= pressure drop per unit length of the filter
q_0	= filtration rate
r	= radius of pore
s	= variable of the probability generating function
t	= time
u	= superficial velocity or the filtration rate
$\mathrm{Var}\{N(t)\}$	= variance of the random variable $N(t)$
V	= cumulative filtrate volume
v	= linear velocity

Greek Letters

λ_n	= intensity of transition
σ	= specific deposit, volume of deposit per unit volume of the filter

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Manuscript received October 27, 1981; revision received July 11, and accepted July 14, 1983.